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# STATIC AND DYNAMIC ANALYSIS OF REED VALVES USING A MINICOMPUTER BASED FINITE ELEMENT SYSTEM

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## ABSTRACT

The purpose of this paper is to present a minicomputer based finite element design procedure and describe its use as a valve design tool. The report consists of a discussion of automatic mesh generation, the finite element method, solution of the static and eigenvalue problems, and two examples where the techniques have been applied to real valve geometries. A comparison of predicted and experimental values of stress, natural frequencies and mode shapes for the two test cases is also included.

## INTRODUCTION

Pressure actuated hermetic compressor reed valves are thin plates made of high strength steel. Their shape is determined by the number and layout of the ports they seal. The dynamic behavior of these components is a function of elastic restoring, inertial body and fluid forces generated during their operation.

The finite element method can be used to perform static and dynamic analyses of reed valves. This method can accommodate any valve geometry and predict deflection, stress and modal characteristics.

The work presented here is an outline of a finite element analysis system used to design and improve reed valves. The difficulties involved in using the method and their solution are discussed. A brief summary of finite element mathematics and solution techniques is included.

## GEOMETRIC MESH GENERATION

Application of the finite element method can be complex and time consuming. Many opportunities exist for the introduction of errors. A few of the more important considerations include:

1. Are the nodes at the correct geometric

locations?

2. Are the nodal connectivities correct?
3. Do the boundary conditions give an accurate portrayal of the component in its operating environment?
4. Are there sufficient elements in regions where the stress being modeled is changing rapidly?

Automatic mesh generation programs can be used to alleviate these types of errors. The approach described here divides the component into regions which are automatically subdivided into triangles or quadrilaterals (1). Figures 1 and 2 demonstrate this discretizing process.

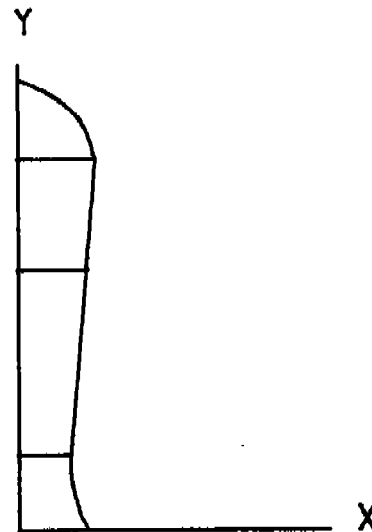


Figure 1 Reed Valve Regions

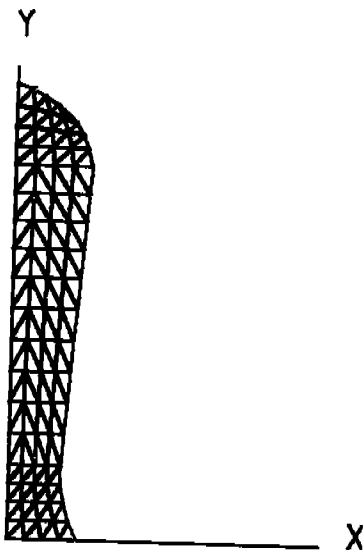


Figure 2 Discretized Reed Valve

#### FINITE ELEMENT METHOD

The element stiffness matrix  $[K_e]$  and equivalent nodal load vector  $\{F_e\}$  as derived by the minimum potential energy theorem appear as follows:

$$[K_e] = \int_{vol} [B]^T [D] [B] DV,$$

$$\{F_e\} = \int_{vol} [N]^T P DV \text{ (body forces),}$$

$$\{F_e\} = \int_{surface} [N]^T Q DS \text{ (surface tractions),}$$

where  $[B]$  is a matrix which maps nodal degrees of freedom to interior strains,

$[D]$  is a matrix which maps interior strains to interior stresses or interior moments in the case of a plate

$[N]$  is a matrix of shape functions which maps nodal degrees of freedom to interior degrees of freedom,

$P$  is a vector of nodal magnitudes of the body forces,

$Q$  is a vector of nodal magnitudes of the surface tractions.

The element mass matrix as derived through the finite element kinetic energy expression is shown below:

$$[M_e] = \int_{vol} [N]^T \rho [N] DV$$

where

$\rho$  = mass density

Details of these derivations may be found in reference (2).

The element mass and stiffness matrices used in this work were developed simultaneously by Argyris (3), Bell (4), and Cowper et al (5). The element is an eighteen degree of freedom plate bending triangle with the following degrees of freedom at each node: deflection ( $w$ ), slopes ( $\partial w / \partial x$ ,  $\partial w / \partial y$ ), curvatures ( $\partial^2 w / \partial x^2$ ,  $\partial^2 w / \partial y^2$ ) and twist ( $\partial^2 w / \partial x \partial y$ ).

The global mass  $[M_g]$  and stiffness  $[K_g]$  matrices are assembled from the element mass and stiffness matrices. The global force vector  $\{F_g\}$  is created from the element equivalent nodal loads in a similar manner.

#### STATIC SOLUTION

Operating stress levels are an important concern in the design of a valve. These levels are predicted by solving the statics problem subject to the imposed boundary conditions and applied loads. The static solution is of the following form:

$$\{F_g\} = [K_g] \{x_g\}$$

where  $\{x_g\}$  represents a vector of unknown nodal displacements. The nodal displacements are found by inverting the stiffness matrix and multiplying by the force vector.

$$\{x_g\} = [K_g]^{-1} \{F_g\}$$

A full inversion and multiplication requiring  $N^3$  and  $N^2$  operations respectively (where  $N$  is the order of the  $[K_g]$  matrix) is not necessary. The problem can be solved by using a form of Gaussian elimination called Cholesky decomposition, followed by forward and backward substitution which uses only  $N^3/3$  and  $4N$  operations respectively (6). Element stresses can be calculated once the nodal displacements are known.

Two programs are being used to solve the statics problem at Tecumseh Products. The first is an 'in-core' solver (one in which the entire  $[K_g]$  matrix is solved in main memory). As a result of the highly banded nature of the  $[K_g]$  matrix this program utilizes the 'skyline' (7) or variable bandwidth storage scheme. The technique stores only non-zero coefficients of the  $[K_g]$  matrix with exception of the zeroes residing within the bandwidth. The second program is an 'out of core' solver (one in which the Gaussian elimination is carried out with only part of the  $[K_g]$  matrix in main memory). This allows large problems to be solved on a relatively small minicomputer. Structures with as many as 800 degrees of freedom have been solved with less than 32K words of memory using this algorithm.

## EIGENVALUE SOLUTION

The following generalized eigenvalue problem results whenever the applied forcing function is periodic in nature.

$$\omega^2 [M_g] \bar{X} = [K_g] \bar{X}$$

The solution of this equation yields natural frequencies ( $\omega$ ) and mode shapes ( $\bar{X}$ ).

Iterative solution techniques were chosen because they efficiently predict dominant frequencies while preserving the banded nature of the problem. Two programs were written for the eigensolution. The 'in-core' eigensolver uses the simultaneous iteration scheme presented by Jennings (8), whereas the 'out of core' solver employs the subspace iteration method of Bathe (7).

The calculated natural frequencies and mode shapes may also be used as input data to a valve simulation. Through the use of the mode superposition method and a compressor simulation, (9, 10) an accurate valve motion and stress history can be calculated.

## EXAMPLES

Two examples are presented to demonstrate the accuracy of the finite element technique.

The first example is the stress analysis of a ring-type reed valve. Figure 3 shows the finite element mesh. Due to symmetry about the x and y axes it was necessary to model only one quarter of the valve. The edge of the tab is simply supported. A prescribed deflection of 0.082 in. was enforced along nodes 56 through 60 to simulate contact with a step relief on the edge of the cylinder bore. The maximum principal stress contours are displayed in figure 4. An experimental stress of 78000 psi was found at location 1 (figure 4) using strain gages.

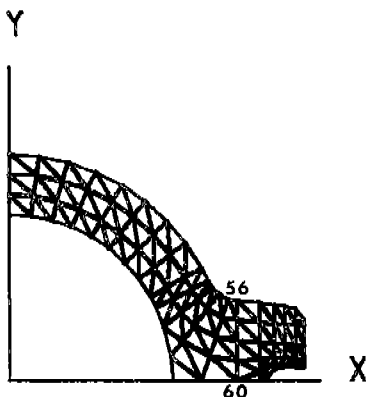


Figure 3 Ring Valve Mesh

CONTOUR	STRESS
A	60000.
B	60000.
C	40000.

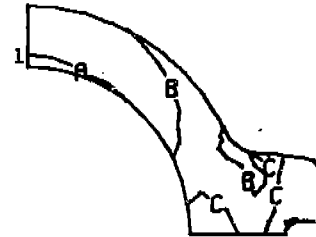


Figure 4 Stress Contours

The second example is the dynamic analysis of a cantilever reed valve. Because of symmetry about the Y axis, only half the valve was modeled. The finite element model and the mode shape corresponding to the fifth natural frequency are shown in figure 5. Figure 6 shows the experimental nodal lines for the fifth mode. Natural frequencies and mode shapes were determined using sinusoidal electromagnetic excitation and a narrow band analyzer. The nine lowest predicted modes and the corresponding experimental values are listed in Table I.

Mode Number	Natural Frequency (Hz.)	
	Predicted	Experimental
1	110	110
2	468	478
3	776	830
4	1432	1536
5	2095	2213
6	3094	3102
7	3321	3574
8	4076	4362
9	4920	5064

Table 1 Natural Frequencies of a Cantilever Valve

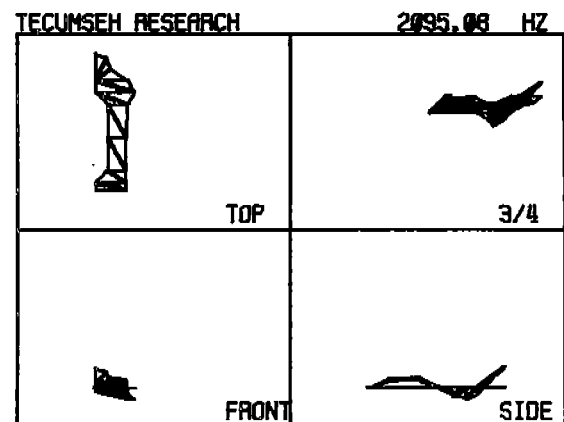


Figure 5 Cantilever Reed Valve

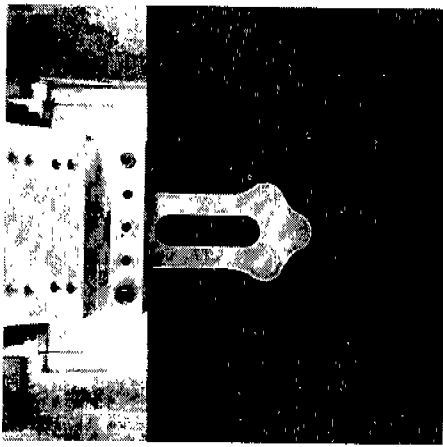


Figure 6 Experimental Mode Shape

#### CONCLUSION

It is not the intention of the authors to claim originality for the finite element method nor any of the various associated solution techniques, but rather to show the application of the method in a systematic manner. The design system presented here operates on a minicomputer and may be used to solve the static and eigenvalue problems. It should also be noted that the finite element method may be used to analyze many other compressor components and is not limited to structural mechanics.

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